**Relationship between Planck and Cosmological Constants**

Einstein and Stern in a paper published in 1913 were the first to quantify zero point energy. They proposed that Planck’s formula for black body radiation be modified by the addition of the quantity, , to better fit the data. Zero point energy is therefore an experimental result.

The Planck’s constant in the expression for zero point energy should be interpreted as a property of the spacetime metric itself. It has the dimensions of action and can therefore be written as,

---------------------------------------------------------------------------------------(1)

where, is the Lagrangian density.

Since the action represented by, *h* is the minimum, irreducible action that any particle must undergo and is independent of the properties of the particle, it must be a property of the spacetime metric itself.

Therefore, the Einstein–Hilbert action in (N) dimensions must be modified to include the above term, yielding,

------------------------------------------------- (2)

Then, by principle of stationary action,

-------------------- (3)

----------------------------- (4)

The left hand side of (4) represents the variation of the metric. It can be shown that,

-------------------------------------------------------------- (5)

------------------------------------------- (6)

Similarly,

----------------------------------------(7)

since, is spatially invariant, .

Substituting, (5), (6) and (7) into (4), we get,

Comparing the above expression with the field equation,

,

allows identification of the cosmological constant with the Lagrangian density;

-----------------------------------------------------------------------------------------(10)

Substituting (10) in eq. (1), we get,

-----------------------------------------------------------------------------------(11)

Assuming that, , is a constant, with a value determined from cosmological observations to be, m-2,

, where, Kg m3 s-2

With, J s, Kg m3 s-2 and m-2, the volume integral evaluates to,

Eq. (11) indicates that when volume of action is , zero point energy effects can no longer be ignored.

Eq. (11) also indicates that the value of is scale dependent; At smaller scales it’s value must increase (to keep constant. This implies that at very short scales, must take on very large values, perhaps resolving the “worst prediction in physics”.

One way to understand the increasing value of at short distances is to assume that Planckton energy distributed along the compactified dimensions becomes important. Since the volume is proportional to rn, where, n is the number of dimensions, at short scales, the value of is increased by the factor of (n-3) over it’s expected value in 3 dimensions.